# Learning Mathematics through Scientific Contents and Methods

Astrid Beckmann
University of Education Schwäbisch Gmünd, Germany astrid.beckmann@ph-gmuend.de

**Abstract**

The basic idea of this paper is to outline a cross-curricular approach between mathematics and science. The aim is to close the often perceived gap between formal maths and authentic experience and to increase the students' versatility in the use of mathematical terms. Students are to experience maths as logical, interesting and relevant through extra-mathematical references. Concrete physical or biological correlations may initiate mathematical activities, and mathematical terms are to be understood in logical contexts. Examples: physical experiments can lead to a comprehensive understanding of the concept of functions and of the intersection of medians in triangles. Biological topics can lead to the concepts of similarity and proportion as well as to the construction of pie charts. In the European ScienceMath Project a variety of teaching modules was developed and tested in secondary schools.

## Background

The European ScienceMath Project is a co-operative project between universities and schools in Germany, Denmark, Finland and Slovenia (coordination: University of Education Schwäbisch Gmünd, Germany; www.sciencemath.ph-gmuend.de). Its central objective is the development and testing of teaching modules for the promotion of mathematical literacy. A cross-curricular approach is used in the natural sciences, in particular in physics, but also in biology and chemistry. Through extra-mathematical references, students are to experience maths as appropriate, meaningful and interesting. The learning in meaningful contexts is designed to contribute to an intuitive mathematical understanding. The idea is, on the one hand, to close the gap between formal maths and authentic experience through the use of contexts and methods taken from the natural sciences (Kaput 1994), and, on the other hand, to allow students to experience the versatility of mathematical terms (Michelsen & Beckmann 2007). That way, mathematical content can be learned in realistic and meaningful contexts and the students' sense of reality can be increased through mathematical insight. In the following, four teaching modules will be presented that were developed at the University of Education Schwaebisch Gmuend. These are concerned with promoting the acquisition of mathematical terms through physical experiments in the fields of the function and the intersection of the median in triangles (centre of gravity) as well as promoting students' competences in the fields of similarity/geometrical proportions and in the construction of pie charts in biological contexts.

## Promoting the acquisition of the Term Function through Physical Experiments

Experiments are well known as methods used in the natural sciences. In maths teaching, they can become part of a new form of instruction. Due to my own extensive experiences and trials in teaching, they are recommendable here, too. There is a lot to be said for experiments in connection with functions because

- the activities in an experiment correspond to the respective aspects of the functional concept: Collecting the individual data equals the aspect of correspondence; collecting the data of a complete series of measurements corresponds to the discrete co-variation, respectively the process level, and transferring the data to a graph leads to continuous co-variation and the object level (Malle 2000, Vollrath 1978, Dubinsky & Harel 1992, Swan 1982, cf. also House of Functional Thinking, Hoefer 2008).
- in carrying out experiments, essential aspects of functional terms are actively experienced. Example: the idea of continual co-variation is experienced, when observing an experimental car that is in motion and continually increases the distance travelled (with stop watch). The object idea, such as anti-proportionality, can be experienced by pumping up a tire with a closed bicycle pump: in reducing the air volume inside the pump, one simultaneously notices an increase in the air pressure.
- experimenting addresses various objectives simultaneously (data collection, functional context, modelling) and allows students to gain experience in various relations to reality (cross-curricular context, every-day experience and concrete quantities, instead of x and y).

Using physical experiments, to promote the understanding of functions, was once again motivated by the results of numerous tests and international comparative studies in the past years and decades, all showing that many students have only a limited understanding of the term function (i.e. Vinner & Dreyfus 1989, PISA-consortium 2004, Beckmann 1999). A special deficit here concerns the inability to interpret graphs correctly and to recognize the functional connection between two variables. It is here that experiments offer a particular opportunity to demonstrate, experience and talk about this functional connection in a concrete way (Beckmann 2006).

Using physical experiments, to promote the understanding of functions (and also of the term variables), was meanwhile tried out with 400 high school students between the ages of 13 and 17 at various school levels (cf. Hoefer & Beckmann 2009, Beckmann 2006, 2007, Zell 2008, Beckmann & Litz 2009). The emphasis here was on experiments, made up of simple materials, to ensure an uncomplicated implementation in maths teaching. However, there were, in addition, cross-curricular tests, in which the reference to maths as well as to physics was discussed (Lukoscheck 2005, Haas & Beckmann 2008). In class, the students carried out the experiments by themselves in groups. They were either observed directly or via video and audio. In addition, the worksheets, that were prepared for each experiment, were evaluated (Beckmann 2006).
The important results, made in these trial runs, were, that the reflection on the data sheets and the discussion of the graphs stimulated the co-variation aspect as well as the formulation of hypotheses in relation to reality, on the background of the co-variation aspect. Furthermore, presentational activities, such as the data's relation to a co-ordinate system, respectively, modelling activities in general, were stimulated. In their descriptions, the students partly used the link with reality and the model simultaneously: "In an additionally plotted graph we were able to see exactly how much light there was at the beginning of the tunnel and how the light intensity was gradually reduced". After carrying out the experiments, many students were able to interpret the graphs. The following graph (figure 1), for example, was no longer interpreted as "The car is rolling downhill" but as "The car is stopping at a crossroads.", "The car had to brake because of an obstacle." "It is in a traffic jam.".

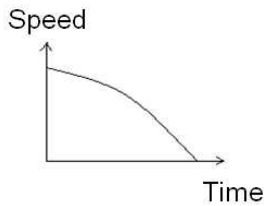


Figure 1
Functional correlation between speed and time

**Point of Intersection of the Medians in a Triangle and the Term Physical Centre of Gravity**

In connection with the topic "Special Points in a Triangle", experiments enable a more global view and more global links, in that the point of intersection of the medians in a triangle appears as only one of the numerous examples for the centre of gravity in bodies. The basic idea of the realisation in class is to enable students to experience the term centre of gravity by placing it in real situations, respectively, in natural science contexts and through experimental activities. All known methods for determining the centre of gravity, such as the hanging or weighing methods, (Heine & Prommersberger 1999) result from the definition of the point of gravity as the point of intersection of all lines of gravity. In the hanging method, the given bodies are hung up in at least two different positions, the lines of gravity are marked and their point of intersection is determined (graph 2). The rule to be learned from this is, that the point of intersection of the medians in a triangle is its centre of gravity and that all axes of symmetry are lines of gravity.
In station work in secondary I, sheets of cardboard and bodies with fixtures for hanging (from the geometry collection or at random composed of Duplo blocs) are made ready for the hanging and weighing methods. The stations stimulate students on the one hand to find the points of gravity experimentally and on the other hand to establish them graphically and check them experimentally. There is also a dynamic geometry system, available for the latter, with the help of which the quality of the median's intersection point in a triangle can be established. Printing the triangles and sticking them onto cardboard reveals the connection with the point of gravity.
In the station work at secondary II level, the analytical description of the points of gravity is made additionally. As an example, the following applies for the point of gravity S in a triangle ABC with the position vectors of S: $\vec{s} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$, which leads us to the interesting question whether that correspondingly applies to the quadrangle $\vec{s} = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c} + \vec{d})$. The result of the hanging methods is, that the validity of the equation depends on the distribution of the mass in the quadrangle. In a full trapeze,

e.g., this results in a different value from that of a “hollow trapeze with an uneven mass distribution at the corners” (figure 2).

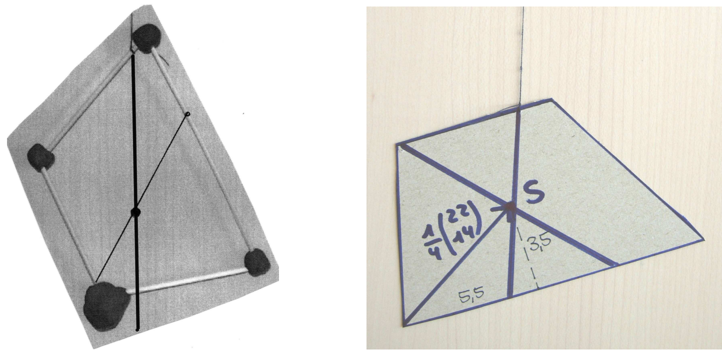


Figure 2
Definition of the point of gravity with hanging method,
Left: hollow trapeze with “heavy corner”,
co-ordinates of the point of gravity S (4.8/4.2).
Right: cardboard trapeze, same size, point of gravity (5.5/3.5).

The difference in the position of the point of gravity in quadrangles of similar size but a different distribution of the weight, can be confirmed mathematically. For the trapeze in figure 2, with corners weighted by plasticine of the masses $m_1=m_2=m_3=0.1$gr. and $m_4=0.2$ gr., the point of gravity works out as:

$$\vec{s} = \frac{1}{0{,}1+0{,}1+0{,}1+0{,}2}\left(0{,}1\begin{pmatrix}0\\0\end{pmatrix}+0{,}1\begin{pmatrix}12\\0\end{pmatrix}+0{,}1\begin{pmatrix}8\\7\end{pmatrix}+0{,}2\begin{pmatrix}2\\7\end{pmatrix}\right)$$

$$=\frac{1}{0{,}5}\left(\begin{pmatrix}1{,}2\\0\end{pmatrix}+\begin{pmatrix}0{,}8\\0{,}7\end{pmatrix}+\begin{pmatrix}0{,}4\\1{,}4\end{pmatrix}\right)=2\cdot\begin{pmatrix}2{,}4\\2{,}1\end{pmatrix}=\begin{pmatrix}4{,}8\\4{,}2\end{pmatrix}$$

In further tasks, corresponding observations of bodies, such as pyramids, are stimulated. A linked procedure is characteristic of this approach, in that the point of gravity is worked out experimentally with the help of the hanging method of real bodies and also mathematically using analytical geometry and integral calculus. A three-dimensional representation and an examination of the point of gravity on the computer, e.g. with the help of Mathematica, enrich the overall experience of the learning process (Cf. learning material at www.sciencemath.ph-gmuend.de).

**Similarity and geometrical Proportions in biological Contexts**

The topic “similarity” plays a central role in maths instruction. A definition can, e. g., be formulated with the help of centric dilation, in that by the term similar figures those figures are understood, that mutually overlap through centric dilation. The original figure and the centrically dilated figure correlate in length, surface and volume, and this can be expressed by the relevant dilation factor k, respectively $k^2$, $k^3$. In biology, there is no similarity in a mathematical sense. Biologists rather speak of allometry, which describes the differences in the proportions (e.g. of organs and limbs) of similar looking animals. By comparing the volume m or the measurements of live animals or their photographs, allometric features can be ascertained by assessing the proportions of externally similar animals, such as domestic cats and wild cats, white and black rhinos, pups and fully grown animals. Example:

$$\frac{m_{tiger}}{m_{domestic\,cat}}=\frac{100kg}{5kg}=20$$

The dilation factor is therefore: $k=\sqrt[3]{20}=2{,}7$. Their shoulder heights (acromion) relate as 1: 2.7, while their surfaces relate as 1: 7.3. Allometries have their biological reasons, such as the different functions of internal organs or the “puppy scheme”, according to which young animals have a bigger head, in order to look more appealing and thereby stimulate a protective behaviour in the parents.

The relationship between surface and volume is also of particular importance. In maths teaching, an important objective is to realize that bodies of the same volume may have totally different surfaces. This can, e. g., be shown with the help of a cube of 1m edge length. If one changes its original dimensions of 1:1:1 edge length to 1:1:2 and then to 1:1:4 etc., retaining the same volume, this leads to a continuous elongation of the cuboids. The biological consequences of the relationship between volume and surface show in the specific body shapes and forms of behaviour of the animals. Dragonflies, e. g., have a slim long body to be able to fly fast, while a beetle of the same volume is rather round and compact. Small endotherms such as field mice and hummingbirds must constantly consume high energy food, in order to be able to equalize the unfavourable ratio between volume and surface (Glaeser 2004).

In the teaching module, these relationships are worked out individually with the help of worksheets (Cf. worksheets at www.sciencemath.ph-gmuend.de). From a mathematical point of view, this is all about deepening the understanding of the topic "similarity - centric elongation" and the "relationships between surface and volume". It is overall about proportions in geometry and the animal kingdom, at which the networking between biology and mathematics takes centre stage: On the one hand, the mathematical relationships are extended in an important way through the biological perspective, and on the other hand, mathematics helps to discover biological phenomena and to understand the biological consequences.

**Pie Chart, Percentage Calculation and Nutrition Circle**

The special characteristic of this teaching module on "healthy eating" is the cross- curricular approach between mathematics and biology, and the teaching about a highly topical issue: more and more children eat the wrong things and are overweight (KIGGS, Kurth & Schaffrath 2007). They often don't even know what is meant by healthy eating. The nutrition circle of the German Nutrition Association (Deutsche Gesellschaft für Ernährung e.V.) (figure 3) can be of support here and forms the starting point and the basis for the teaching module to be introduced (Grube 2008). It informs about healthy eating in a clearly laid out circular diagram, in which the individual segments represent food categories, such as cereals, fruit, milk products and meats. The size of each segment gives an indication of the respective salutary amount.


Figure 3 Nutrition Circle of the German Nutrition Association (DGE 2005)

From a mathematical point of view, the teaching module serves the purpose of introducing circular diagrams (including the application of percentage calculation); from a biological point of view it serves to stimulate healthy eating. The module starts with a homework task that the students have to do before the start of the sequence: They are asked to weigh all the foods that they eat in a (if possible) normal day and to note down and order the quantities according to the categories of the nutrition circle. They enter these in a chart. In the first lesson, the nutrition circle is introduced. It serves as an impulse and to initiate a discussion, based on the question: Are my eating habits in accordance with the nutrition plan of the nutrition circle? For comparison, one's own eating habits are entered into a circular diagram. This is achieved by calculating the percentages and transferring them to angular measures. A further task is establishing one's own perfect nutrition plan that takes into account the parameters of the DGE. This requires a form of thinking in which percentage calculation, circular diagrams and nutritional recommendations are directly linked.

This teaching module was tried out in a German secondary I school in the school year 2007/2008 with students aged 14 – 16 (Grube 2008). The students were highly motivated and were even stimulated to talk about nutrition into the breaks. The topic and the tasks constantly led them to linked thinking between biology and mathematics. The need to reflect on the procedure of entering data in the circular diagram, in order to compare their own eating habits with those suggested by the DGE, proved to achieve excellent motivation. The comparison of nutrition plans among class mates led to exited discussions, and students worked out independently, that in spite of a higher percentage value a lower total percentage can be achieved, as the basic value is the decisive factor.

## Literature


Beckmann, A. (2006a). Experiments for learning the concept of function. Experimente zum Funktionsbegriffserwerb. Köln (Aulis)

Beckmann, A. (2007). Non-linear functions in secondary school of lower qualification level (German Hauptschule). The Montana Mathematics Enthuisiast, vol, no 2, June 2007

Beckmann, A. (1999): The concept of function.. Der Funktionsbegriff als Unterrichtsgegenstand zu Beginn des Mathematikunterrichts in der zweijährigen Höheren Berufsfachschule. In: Journal für Mathematikdidaktik (JMD) 20/4, S.274-299

Beckmann & Litz (2009). Non-linear functions. Nicht-lineare Funktionen – ein Beitrag zur Förderung von mathematical literacy in der Hauptschule. Sammelband Hauptschulforschung. Forschungsverbund Hauptschule 2009

Dubinsky, E. & Harel, G. (Ed.) (1992). The Concept of Function. Aspects of Epistemology and Pedagogy. Mathematical. Ass. of America.

Glaeser, G. (2004). The mathematical toolbox. Der mathematische Werkzeugkasten. Anwendungen in Natur und Technik. München (Spektrum Akademischer Verlag)

Grube, A. (2008). The DGE-Nutrition Circle as a topic in cross-curricular mathematical lessons. Der DGE-Ernährungskreis als Thema im fächerübergreifenden Mathematikunterricht – Entwicklung und Erprobung einer Unterrichtssequenz unter besonderer Berücksichtigung von Anwendungsaufgaben zur Prozentrechnung. In: Beckmann, A. (Hg.). Ausgewählte Unterrichtskonzepte im Mathematikunterricht in unterrichtlicher Erprobung. Bd 5. Berlin (Franzbecker). 89-108

Haas, B. & Beckmann, A. (2008). Doing physical experiments, mathematical modelling, interdisciplinary working. Physikalisches Experimentieren, mathematisches Modellieren, interdisziplinäres Arbeiten. In: Beckmann, A. (Hg.). Ausgewählte Unterrichtskonzepte im Mathematikunterricht in unterrichtlicher Erprobung. Bd 5: Hildesheim, Berlin (Franzbecker), 13-48

Heine, A. & Prommersberger, H. (1999). Physics and technics. Physik und Technik. Hamburg (Verlag Handwerk und Technik)

Höfer, Th. (2008). House of functional thinking. Das Haus des funktionalen Denkens – Entwicklung und Erprobung eines Modells für die Planung und Analyse methodischer und didaktischer Konzepte zur Förderung des funktionalen Denkens. Hildesheim, Berlin (Franzbecker)

Höfer, Th. & Beckmann, A. (2009). Supporting mathematical literacy: examples from a cross-curricular project”. ZDM 41/1 (2009), 223

Kaput, J. (1994). The Representational Roles of Technology in Connecting Mathematics with Authentic Experience. Biehler, R. et al. (Hg.). Didactics on Mathematics as a Scientific Discipline. Dodrecht (Kluwer Academic Publichers). 379-397

Lukoschek, M. (2005). Functuional thinking with experiments. Funktionales Denken mit physikalischen Experimenten – ein fächerübergreifender Unterrichtsversuch zwischen Mathematik und Physik. Wiss. Hausarbeit, Institut für Mathematik und Informatik, Pädagogische Hochschule Schwäbisch Gmünd

Malle, G. (2000). Two aspects of the concept of variable: correspondence and covariation. Zwei Aspekte des Variablenbegriffs: Zuordnung und Kovariation. In: Mathematik Lehren 103, 8-11

Michelsen, C. & Beckmann, A. (2007). Supporting the concept understanding through expanding the domain. Förderung des Begriffsverständnisses durch Bereichserweiterung. In: MU ½/ 53, 45-57

PISA-Konsortium (Hrsg.) (2004). PISA 2003. The Educational background of the youth. Der Bildungsstand der Jugendlichen in Deutschland – Ergebnisse der zweiten internationalen Vergleichsstudie. Münster (Waxmann-Verlag).

Swan, M. (1982). The teaching of functions and graphs. In: Conference on functions 1-5. Enschede, 151-165

Vinner, S. & Dreyfus, T. (1989): Images and Definitions for the Concept of Function. Journal Research in Mathematical Education 20 4, 356-366

Vollrath, H.-J (1978). Experiments supporting the concept of function. Schülerversuche zum Funktionsbegriff. Der Mathematikunterricht 4, 90-101

Zell, S. (2008). Investigating the concept of variable through physical experiments. Erkunden des Variablenbegriffs mit Hilfe physikalischer Experimente. In: Beckmann, A. (Hg.). Ausgewählte Unterrichtskonzepte im Mathematikunterricht in unterrichtlicher Erprobung. Bd 5: Hildesheim, Berlin (Franzbecker). 49-87